
ADDITIONAL MATHEMATICS**4037/13**

Paper 1

October/November 2017

MARK SCHEME

Maximum Mark: 80

Published

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This document consists of **10** printed pages.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	Using $\tan^2 \theta + 1 = \sec^2 \theta$ to obtain $y = 2(\tan^2 \theta + 1)$ or $(x + 5)^2 = \sec^2 \theta - 1$ $(x + 5)^2 + 1 = \frac{y}{2}$	M1	use of correct identity
	$y = 2((x + 5)^2 + 1)$ oe	A1	
2	$\frac{dy}{dx} = 10e^{5x} + 3$ an attempt at integration in form $ae^{5x} + bx$	M1	
	$y = \frac{10}{5}e^{5x} + 3x (+c)$	A1	condone omission of c
	attempt to find c using $x = 0, y = 9$	M1	M1dep
	$y = 2e^{5x} + 3x + 7$	A1	
3	$9 < 4k(k - 4)$ $4k^2 - 16k - 9$	M1	use of the discriminant with correct values
	$(2k - 9)(2k + 1)$	M1	M1dep for solution of <i>their</i> quadratic to obtain critical values
	Critical values $\frac{9}{2}, -\frac{1}{2}$	A1	
	$k < -\frac{1}{2}, k > \frac{9}{2}$	A1	
4	$a = 3$	B1	
	$b = 8$	B1	
	$\frac{5}{2} = 3 \cos\left(8 \times \frac{\pi}{12}\right) + c$	M1	substitution of $x = \frac{\pi}{12}$ and $y = \frac{5}{2}$ to find c
	$c = 4$	A1	
5(i)	$\frac{5}{14}(7x - 10)^{\frac{2}{5}}$	B2	B1 for $k(7x - 10)^{\frac{2}{5}}$

Question	Answer	Marks	Guidance
5(ii)	$\frac{5}{14} \left[(7x-10)^{\frac{2}{5}} \right]_6^a = \frac{25}{14}$ $\frac{5}{14} (7a-10)^{\frac{2}{5}} - \frac{5}{14} (7 \times 6 - 10)^{\frac{2}{5}} = \frac{25}{14}$ $(7a-10)^{\frac{2}{5}} - 4 = 5$	M1	correct application of limits for $k(7x-10)^{\frac{2}{5}}$
	$a = \frac{9^{\frac{5}{2}} + 10}{7}$	M1	M1dep for evaluation of $(7 \times 6 - 10)^{\frac{2}{5}}$ and correct order of operations to find a , including dealing with power.
	$a = \frac{253}{7} \text{ or } 36\frac{1}{7}$	A1	
6(i)	Gradient = $\frac{2.4-0.9}{0.2-0.8} (= -2.5)$	B1	
	$\ln y = -\frac{5}{2}x^2 + c$	M1	straight line form and correct substitutions to find c
	$\ln y = -\frac{5}{2}x^2 + 2.9 \text{ oe}$	A1	
	<u>Alternative method</u> $2.4 = p(0.2) + q$ $0.9 = p(0.8) + q$	B1	
	Correct method of solution to find p and q from two correct equations	M1	M1dep
	$\ln y = -\frac{5}{2}x^2 + 2.9$	A1	
6(ii)	$y = e^{\left(-\frac{5}{2}x^2 + 2.9\right)}$	M1	dealing with ln
	$y = e^{-\frac{5}{2}x^2} \times e^{2.9}$	M1	M1dep for dealing with the index
	$y = 18.2z^{-\frac{5}{2}}$	A1	

Question	Answer	Marks	Guidance
7(i)	$64 - 48x^2 + 15x^4$	B3	B1 for each correct term in final line of response
7(ii)	$(64 - 48x^2 + 15x^4) \left(\frac{1}{x^2} + 2 + x^2 \right)$	B1	B1 for $\frac{1}{x^2} + 2 + x^2$ oe
	at least two correctly obtained products leading to terms in x^2	M1	
	Term in x^2 : $64 + 15 - 96$	A1	FT for correct evaluation of <i>their</i> $64 + (2 \times \text{their} - 48) + \text{their } 15$
	$= -17$	A1	
8(i)	attempt to differentiate a product	M1	
	$\frac{dy}{dx} = \left((x-4) \times \frac{5}{3} \times 3(3x-1)^{\frac{2}{3}} \right) + (3x-1)^{\frac{5}{3}}$	A2	A1 for (+) $\left((x-4) \times \frac{5}{3} \times 3(3x-1)^{\frac{2}{3}} \right)$ A1 for (+) $(3x-1)^{\frac{5}{3}}$
	$= (3x-1)^{\frac{2}{3}} ((5x-20) + (3x-1))$	M1	use of $(3x-1)^{\frac{5}{3}} = (3x-1)^{\frac{2}{3}} (3x-1)$
	$= (3x-1)^{\frac{2}{3}} (8x-21)$	A1	
8(ii)	When $x = 3$, $\frac{dy}{dx} = 8^{\frac{2}{3}} \times 3$	M1	$(3 \times 3 - 1)^{\frac{2}{3}} \times k$ or $(9 - 1)^{\frac{2}{3}} \times k$ or $4 \times k$ (where k is any number)
	$\partial y = 8^{\frac{2}{3}} \times 3 \times h$	M1	M1dep for <i>their</i> $\left((9 - 1)^{\frac{2}{3}} \times k \right) \times h$
	$\partial y = 12h$	A1	
9(a)(i)	720	B1	
9(a)(ii)	240	B1	
9(a)(iii)	$k \times 4! \times 2$ or $240 - k \times 4! \times 2$ or correct equivalents with no extra terms added or subtracted	B1	
	$4 \times 4! \times p$ or correct equivalents with no extra terms added or subtracted	B1	
	192	B1	

Question	Answer	Marks	Guidance
9(b)(i)	6435	B1	
9(b)(ii)	With twins: ${}^{13}C_6$ or 1716 Without twins: ${}^{13}C_8$ or 1287	B2	B1 for ${}^{13}C_6$ or 1716 or ${}^{13}C_8$ or 1287 B1 for (${}^{13}C_6$ and ${}^{13}C_8$) or (1716 and 1287) with no multiples and no extra terms
	Total: $1716 + 1287 = 3003$	B1	3003 from a correct method
10(a)	matrix multiplication, must have at least 2 correct elements	M1	
	$\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 2a - 5b & 3a + 4b \end{pmatrix}$	A1	
	$2a - 5b = 18$ $3a + 4b = 4$	M1	formation and solution of simultaneous equations
	leading to $a = 4, b = -2$	A1	
	<u>Alternate scheme</u> $\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix}$ $\mathbf{ABB}^{-1} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix} \mathbf{B}^{-1}$	M1	Correct plan
	Correct inverse	B1	
	$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ a & b \end{pmatrix} = \frac{1}{23} \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 5 & 2 \end{pmatrix}$	M1	Correct order and method of multiplication with at least two correct elements
	leading to $a = 4, b = -2$	A1	
10(b)(i)	$-\frac{1}{17} \begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$ oe	B2	B1 for $-\frac{1}{17}$ B1 for $\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$
10(b)(ii)	$\mathbf{Z} = -\frac{1}{17} \begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 4 & 0 \end{pmatrix}$	M1	pre-multiplication with two elements correct
	$= -\frac{1}{17} \begin{pmatrix} 19 & 2 \\ 8 & 8 \end{pmatrix}$ oe	A2	A1 for four correct of $-\frac{1}{17}, 19, 2, 8, 8$

Question	Answer	Marks	Guidance
11(i)	1.48	B1	
11(ii)	$\frac{1}{2} \times 10^2 \times \theta = 21.8$	M1	correct use of sector area
	$\theta = 0.436$	A1	
11(iii)	$\angle BOC = \frac{2\pi - 1.48 - 0.436}{2} \quad (= 2.18(4))$	B1	2.18(4) or unsimplified
	$BC = 20 \sin\left(\frac{1}{2} \angle BOC\right)$ or $BC = \frac{10 \times \sin BOC}{\sin\left(\frac{\pi - BOC}{2}\right)}$ or $BC = \sqrt{(200 - 200 \cos BOC)}$ $BC = 17.7(5)$	M2	M1 for a complete correct method to find BC using <i>their</i> angle BOC M1 for a correct plan using 14.8, <i>their</i> BC and $10 \times$ <i>their</i> answer to (ii)
	Perimeter = $14.8 + (2 \times 17.7(5)) + 4.36$ = 54.7 or 54.6	A1	awrt 54.7 or awrt 54.6

Question	Answer	Marks	Guidance
11(iv)	Area = $\left(\frac{1}{2} \times 10^2 \times 1.48\right) + 21.8 + 2\left(\frac{1}{2} \times 10^2 \sin 2.18(4)\right)$	B2	B1 for $\left(\frac{1}{2} \times 10^2 \times 1.48\right) + 21.8$ B1 for $2\left(\frac{1}{2} \times 10^2 \sin 2.18(4)\right)$
	= 178	B1	awrt 178 from correct working
	<u>Alternative method 1</u>		
	Segment area = $\frac{1}{2}(10^2(2.18 - \sin 2.18))$	B1	B1 for $2 \times \frac{1}{2}(10^2(2.18(4) - \sin 2.18(4)))$
	Area required = $100\pi - 2 \times \frac{1}{2}(10^2(2.18(4) - \sin 2.18(4)))$	B1	
	= 178	B1	awrt 178 from correct working
	<u>Alternative method 2</u>		
	Area of trapezium = $\frac{1}{2}((13.5 + 4.33)(17.1))$	B1	correct area of trapezium <i>ABCD</i> (allow unsimplified)
Area of segments = $\frac{1}{2}(10^2(1.48 - \sin 1.48)) +$ $\frac{1}{2}(10^2(0.436 - \sin 0.436))$	B1	correct area of both segments (allow unsimplified)	
= 178	B1	awrt 178 from correct working	

Question	Answer	Marks	Guidance
12(i)	$2x^2 + 5x - 12 = 0$ or $y^2 + 3y - 28 = 0$	M1	attempt to get in terms of one variable
	$(2x - 3)(x + 4) = 0$ or $(y + 7)(y - 4) = 0$	M1	M1dep for solution of a three term quadratic
	leading to $x = -4, y = -7$ and $x = \frac{3}{2}, y = 4$	A2	A1 for each 'pair'
	Midpoint $M \left(\frac{\frac{3}{2} - 4}{2}, \frac{4 + (-7)}{2} \right) \left(= \left(-\frac{5}{4}, -\frac{3}{2} \right) \right)$	A1	correctly obtained midpoint
	Gradient of $PQ = 2$	B1	may be implied
	Perp gradient = $-\frac{1}{2}$	M1	$\frac{-1}{\text{their gradient of } PQ}$
	Perp bisector: $y + \frac{3}{2} = -\frac{1}{2} \left(x + \frac{5}{4} \right)$	M1	M1dep for equation of perp bisector using <i>their</i> perp gradient and <i>their</i> midpoint. (unsimplified)
	$y = -\frac{1}{2}(-10) - \frac{17}{8} = \frac{23}{8}$ or $\frac{23}{8} = -\frac{1}{2}x - \frac{17}{8} \rightarrow x = -10$	A1	all correct so far and for verification using a correct equation

Question	Answer	Marks	Guidance
12(ii)	$\text{Area} = \frac{1}{2} \times \left(\frac{17}{8} + 1 \right) \times \frac{5}{4}$	M1	finding R , S and RS
	correct method for finding area	M1	M1dep
	$= \frac{125}{64} \text{ or } 1.95 \text{ or } 1\frac{61}{64}$	A1	
	<u>Alternative method 1</u>		
	$\text{Area} = \frac{1}{2} \times \frac{\sqrt{125}}{4} \times \frac{\sqrt{125}}{8}$	M1	finding R , S , RM and MS
	correct method for finding area	M1	M1dep
	$= \frac{125}{64} \text{ or } 1.95 \text{ or } 1\frac{61}{64}$	A1	
	<u>Alternative method 2</u>		
$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & 0 & \frac{-5}{4} & 0 \\ 1 & \frac{-17}{8} & \frac{-3}{2} & 1 \end{vmatrix}$	M1	finding R and S to obtain their $\frac{1}{2} \begin{vmatrix} 0 & 0 & \frac{-5}{4} & 0 \\ 1 & \frac{-17}{8} & \frac{-3}{2} & 1 \end{vmatrix}$	
$= \frac{1}{2} \left -\frac{5}{4} - \frac{85}{32} \right \text{ oe}$	M1	M1dep for correct method of evaluation	
$= \frac{125}{64} \text{ or } 1.95 \text{ or } 1\frac{61}{64}$	A1		